MOTION OF A POLYDISPERSE MATERIAL IN A VERTICAL GAS FLOW

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The method of solution previously presented [1] for a two-phase flow is based on replacing the actual interaction between groups of particles by some continuously acting force. Here a more rigorous method of solution is presented via an analysis of the laws of collision between numerous particle groups. Only steady-state flows under isothermal conditions are considered, to simplify the problem.

§1. Case of a material with two fractions. The speed of a particle of any size takes values in a certain range at each point in the presence of collisions [1], so we

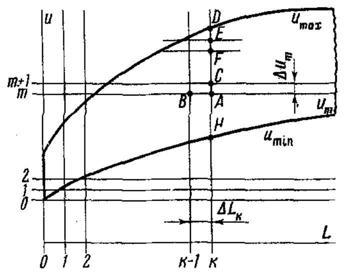

Fig. 1

introduce the velocity-distribution functions for the two fractions,

$$d\beta_i' = f_i(u, L)\,du \qquad (i = 1, 2), \qquad (1.1)$$

in which β_i' is the bulk flow concentration of fraction i, u is particle velocity, L is flow length, the subscript 1 refers to large particles, and the subscript 2 refers to small ones.

We assume that the particles are spherical and move along paths parallel to the axis of the channel [1]. Turbulent pulsations of the gas flow are neglected.

Let the group of small particles at time τ lie at cross section L and have velocities in the range (u_2, $u_2 + du_2$). *

If no particle in the group undergoes a collision in time $d\tau$, then the group at time $\tau + d\tau$ lies in cross section $L + u_2 d\tau$, and the particles have velocities

$$u_2 + a_2 d\tau, \qquad \left[1 + u_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) d\tau\right] du_2 ,$$

in which a is the acceleration of a particle.

However, collisions cause certain particles to leave the group in time $d\tau$, while others enter it; hence the particle concentration in the group at time $\tau + d\tau$ will differ from $f_2(u_2, L)du_2$. Assuming that the increase in particle concentration in the group is proportional to $du_2 d\tau$ for du_2 and $d\tau$ sufficiently small [2], and denoting the coefficient of proportionality by $\partial_e f_2/\partial\tau$, we get

$$\Big\{f_2\Big(u_2 + a_2 d\tau,\; L + u_2 d\tau\Big)\left[1 + u_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) d\tau\right] - f_2(u_2, L)\Big\} du_2 = \frac{\partial_e f_2}{\partial\tau} du_2 d\tau ,$$

or

$$u_2 \frac{\partial f_2}{\partial L} + a_2 \frac{\partial f_2}{\partial u_2} + u_2 f_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) = \frac{\partial_e f_2}{\partial\tau} . \qquad (1.2)$$

The rate of change of f_2 due to collisions may be expressed as the sum of terms owing to collisions with small and large particles, respectively:

$$\partial_e f_2 / \partial\tau = (\partial_e f_2 / \partial\tau)_2 + (\partial_e f_2 / \partial\tau)_1 . \qquad (1.3)$$

Consider groups A and B of small particles, which in section L move with velocities (u_2, du_2) and (u_3, du_3), respectively. The following is the concentration of the group-A particles that collide with group-B particles in time $d\tau$ and so leave the group:*

$$f_2(u_2)\,du_2 \frac{d\tau}{\Delta\tau_{23}}, \qquad \Delta\tau_{23} = \frac{\delta_2 u_3}{6\,|u_3 - u_2|\,w f_2(u_3)\,du_3}, \qquad (1.4)$$

in which $\Delta\tau_{23}$ is the mean time of free motion of a group-A particle between collisions with group-B particles [1], δ is particle size, and w is gas speed.

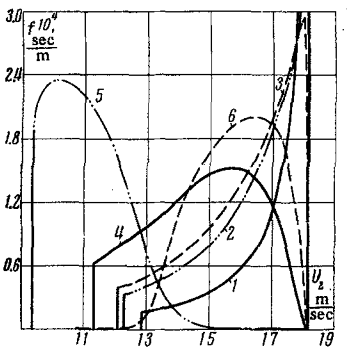

Fig. 2

It follows from (1.4) that the total loss of particles from group A on account of collisions with all small particles is

*For brevity, this is written below as u_2, du_2.

*For brevity, $f(u, L)$ is written as $f(u)$.

$$\frac{6}{\delta_2} w f_2(u_2)\, du_2\, d\tau \int\limits_{u_{2\min}}^{u_{2\max}} f_2(u_3) \frac{|u_3 - u_2|}{u_3}\, du_3 . \qquad (1.5)$$

Let group C of small particles move with speeds (u_4, du_4), this range being chosen such that a group-C particle by collision with a group-B particle acquires a speed in the range (u_2, du_2) and enters group A.

The following is the concentration of group-C particles that collide with group-B particles in time $d\tau$:

$$f_2(u_4)\, du_4\, d\tau / \Delta\tau_{43} . \qquad (1.6)$$

The speed of the group-C particles is defined [1] by

$$u_4 = \frac{4u_2 - (1+k) u_3}{3-k}, \qquad du_4 = \frac{4}{3-k} du_2 , \qquad (1.7)$$

in which k is the coefficient of restitution on collision. The following is the concentration of particles entering group A in time $d\tau$ as a result of collision with small particles, as given by (1.4), (1.6), and (1.7):

$$\frac{96}{(3-k)^2 \delta_2} du_2\, d\tau \;\times$$

$$\times \int\limits_{u_{2\min}}^{u_{2\max}} f_2(u_3) f_2\left(\frac{4u_2 - (1+k)u_3}{3-k}\right) \frac{|u_3 - u_2|}{u_3}\, du_3 . \qquad (1.8)$$

It follows from (1.5) and (1.8) that

$$\left(\frac{\partial_e f_2}{\partial \tau}\right)_2 = \frac{6}{\delta_2} w \int\limits_{u_{2\min}}^{u_{2\max}} \left[\frac{16}{(3-k)^2} f_2\left(\frac{4u_2 - (1+k)u_3}{3-k}\right) - \right.$$

$$\left. - f_2(u_2)\right] f_2(u_3) \frac{|u_3 - u_2|}{u_3}\, du_3 . \qquad (1.9)$$

The rate of change of f_2 from collision with large particles is derived similarly.

Then (1.2) may be put as

$$u_2 \frac{\partial f_2}{\partial L} + a_2 \frac{\partial f_2}{\partial u_2} + u_2 f_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) = \frac{6}{\delta_2} w \;\times$$

$$\times \int\limits_{u_{2\min}}^{u_{2\max}} \left[\frac{16}{(3-k)^2} f_2\left(\frac{4u_2 - (1+k)u_3}{3-k}\right) - \right.$$

$$\left. - f_2(u_2)\right] f_2(u_3) \frac{|u_3 - u_2|}{u_3}\, du_3 +$$

$$+ \frac{3}{2} w \frac{(\delta_1 + \delta_2)^2}{\delta_1^3} \int\limits_{u_{1\min}}^{u_{1\max}} [\sigma_1^2 f_2 (\sigma_1 u_2 -$$

$$- \sigma_2 u_1) - f_2(u_2)] f_1(u_1) \frac{|u_1 - u_2|}{u_1}\, du_1$$

$$\left(\sigma_1 = \frac{2(\delta_1^3 + \delta_2^3)}{(1-k)\delta_1^3 + 2\delta_2^3}, \; \sigma_2 = \frac{(1+k)\delta_1^3}{(1-k)\delta_1^3 + 2\delta_2^3}\right). \qquad (1.10)$$

The equation for f_1 is

$$u_1 \frac{\partial f_1}{\partial L} + a_1 \frac{\partial f_1}{\partial u_1} + u_1 f_1 \frac{d}{du_1}\left(\frac{a_1}{u_1}\right) =$$

$$= \frac{6}{\delta_1} w \int\limits_{u_{1\min}}^{u_{1\max}} \left[\frac{16}{(3-k)^2} f_1\left(\frac{4u_1 - (1+k)u_3}{3-k}\right) - \right.$$

$$\left. - f_1(u_1)\right] f_1(u_3) \frac{|u_3 - u_1|}{u_3}\, du_3 +$$

$$+ \frac{3}{2} w \frac{(\delta_1 + \delta_2)^2}{\delta_2^3} \int\limits_{u_{2\min}}^{u_{2\max}} [\sigma_3^2 f_1 (\sigma_3 u_1 -$$

$$- \sigma_4 u_2) - f_1(u_1)] f_2(u_2) \frac{|u_2 - u_1|}{u_2}\, du_2$$

$$\left(\sigma_3 = \frac{2(\delta_1^3 + \delta_2^3)}{(1-k)\delta_2^3 + 2\delta_1^3}, \; \sigma_4 = \frac{(1+k)\delta_2^3}{(1-k)\delta_2^3 + 2\delta_1^3}\right). \qquad (1.11)$$

It follows from (1.1) that the solutions to (1.10) and (1.11) must obey the following normalization conditions:

$$\int\limits_{u_{i\min}}^{u_{i\max}} f_i(u, L)\, du = \beta_i' . \qquad (1.12)$$

It is assumed that the $f_i(u, L)$ differ from zero in the ranges $(u_{i\min}, u_{i\max})$ and are identically zero outside these ranges. These intervals have to be determined in order to solve (1.10) and (1.11) numerically.

The small particles in the stabilized part of the flow cannot move more rapidly than they do in the absence of the coarse ones, i.e., $u_{2\max}^{(\infty)} = w - v_2$, in which v is particle velocity (the superscript ∞ indicates that the quantity refers to the stabilized part). Similarly, it follows that $u_{1\min}^{(\infty)} = w - v_1$. Here we have

$$u_{2\min}^{(\infty)} = u_{1\min}^{(\infty)}, \qquad u_{1\max}^{(\infty)} = u_{2\max}^{(\infty)} . \qquad (1.13)$$

For example, suppose that $u_{2\min}^{(\infty)} < u_{1\min}^{(\infty)}$. Then in some part of the flow, the aerodynamic resistance and the collisions with large particles cause the speed of all small particles moving at less than $u_{1\min}^{(\infty)}$ to increase at least to $u_{1\min}^{(\infty)}$. If $u_{2\min}^{(\infty)} > u_{1\min}^{(\infty)}$, on the other hand, a small particle with the minimum velocity will collide with some slower large particle and acquire a velocity less than $u_{2\min}^{(\infty)}$, and the next collision will reduce the velocity further, and so on. The flow therefore must contain small particles whose velocity differs by an arbitrarily small amount from $u_{1\min}^{(\infty)}$. The correctness of the second equality in (1.13) may be demonstrated similarly.

As regards the acceleration section, we consider the case where f_1 and f_2 in the inlet section differ from zero in some range $(u_{\min}^{(0)}, u_{\max}^{(0)})$; for concreteness, we assume that $u_{\min}^{(0)} < u_{\min}^{(\infty)}$ and $u_{\max}^{(0)} < u_{\max}^{(\infty)}$ where superscript 0 denotes that the quantity relates to the inlet section. It can then be shown by induction that the equalities of (1.13) apply for all sections.

Let the velocities of the particles of both fractions take values in the range $(u_{\min}^{(A)}, u_{\max}^{(A)})$ in some section A. For any section B further from the inlet than A we have

$$u_{1\min}^{(B)} > u_{\min}^{(A)}, \qquad u_{2\max}^{(B)} > u_{\max}^{(A)} , \qquad (1.14)$$

where the quantities on the left can be deduced from the equations of motion for a monodisperse material [3]. A large particle having the maximum velocity in section A will move [3] with a velocity $u'_{1\max} < u_{2\max}^{(B)}$ in section B, whereas if this particle undergoes sufficient collisions with small particles while moving along part AB, it may at B have acquired a velocity differing by an arbitrarily small amount from $u_{2\max}^{(B)}$ (but not exceeding $u_{2\max}^{(B)}$). Similarly it can be shown that

$$u_{2\min}^{(B)} = u_{1\min}^{(B)}.$$

System (1.10)–(1.11) can be solved by the grid method; Fig. 1 shows the region for determination of the functions and the grid u = constant, L = constant. The curves for u_{max} and u_{min} are derived from the equations of motion for a monodisperse material [3] for small and large particles, respectively. We denote the right-hand sides of (1.10) and (1.11), respectively, by I_1 and I_2; furthermore, we replace the partial derivatives in these equations by the difference relations

$$\frac{\partial f_i}{\partial L} = \frac{f_i(A) - f_i(B)}{\Delta L_k}, \quad \frac{\partial f_i}{\partial u_i} = \frac{f_i(C) - f_i(A)}{\Delta u_m} \quad (i = 1, 2) \quad (1.15)$$

to get

$$f_i(A) = \left(I_i + \frac{u_m}{\Delta L_k} f_i(B) - \frac{a_{im}}{\Delta u_m} f_i(C)\right) \times$$

$$\times \left(\frac{u_m}{\Delta L_k} - \frac{a_{im}}{\Delta u_m} + u_m \frac{d}{du}\frac{a_{im}}{u_m}\right)^{-1} \quad (i = 1, 2) \quad . \quad (1.16)$$

Let f_1 and f_2 be known in section k − 1. The values of the functions at the ends (D and H) are unknown, since the normalization conditions of (1.12) serve as boundary conditions. We put $f_1(D) \equiv \Phi_1$, $f_2(D) \equiv \Phi_2$ and from (1.16) calculate successively $f_i(E)$, $f_i(F)$, ..., $f_i(H)$, which are linear functions of Φ_1 and Φ_2, so Φ_1 and Φ_2 are uniquely determined from (1.12).

It is clear that the desired functions must be known in the inlet section.

This method of solution involves a large volume of computation which must be performed by computer.

§2. Consider the particular case that can be solved approximately; let $\delta_1 \gg \delta_2$. As the mass of a small particle is then negligible relative to the mass of a large one, the velocity change in a large particle due to a single collision with a small one is very slight, but the collisional frequency must be very high [1]. We replace the collisional action of the small particles on a large one by a continuously acting force. All large particles in a given section must move with the same speed U_1. As previously, we assume that the velocities of the small particles take values in some range and consider f_2, the velocity-distribution function for the small particles. However, we simplify the treatment by neglecting the collisions between small particles.

Let k ≠ 1. We neglect the mass of a small particle relative to that of a large one, and instead of the expressions for σ_1 and σ_2 in (1.10) we get

$$\sigma_1 \approx \frac{2}{1-k}, \qquad \sigma_2 \approx \frac{1+k}{1-k}. \quad (2.1)$$

As all the large particles have the same speed, we must have

$$f_1(u_1) = \beta_1' \delta(u_1 - U_1), \quad (2.2)$$

in which δ(t) is the delta function. We use (2.1) and (2.2) with the notation

$$\psi = {}^3/_2\, w \beta_1' (\delta_1 + \delta_2)^2 / \delta_1^3 \quad (2.3)$$

to replace (1.10) by

$$u_2 \frac{\partial f_2}{\partial L} + a_2 \frac{\partial f_2}{\partial u_2} + u_2 f_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) = \psi\left(\frac{u_2}{U_1} - 1\right) \times$$

$$\times \left[\frac{4}{(1-k)^2} f_2\left(\frac{2}{1-k} u_2 - \frac{1+k}{1-k} U_1\right) - f_2(u_2)\right] \quad (k \neq 1). \quad (2.4)$$

If k = 1, the velocity of a small particle after collision is always U_1 if the above assumptions apply [1], and the flow contains no small particle that after collision with a large one has a velocity in the range u_2, du_2. Then the first term in the expression in square brackets in (2.4), which equals (apart from a factor) the rate of change in f_2 on account of entry of new particles into the group u_2, du_2, is identically zero for k = 1, and (2.4) is replaced by

$$u_2 \frac{\partial f_2}{\partial L} + a_2 \frac{\partial f_2}{\partial u_2} + u_2 f_2 \frac{d}{du_2}\left(\frac{a_2}{u_2}\right) =$$

$$= -\psi\left(\frac{u_2}{U_1} - 1\right) f_2 \quad (k = 1). \quad (2.5)$$

We have

$$\partial f_2 / \partial L = 0$$

for the stabilized part of the flow, and (2.5) is readily integrated in this case.

The form of the solution is dependent on the form of the expression for a_2, i.e., on the range of values for R_2 (the Reynolds number) for the motion of the small particles [3]. For instance, the following are solutions of (2.5):

R_2 = 13–800

$$f_2 = C u_2 \frac{(t - \sqrt{v_2})^{2\varepsilon - 1}}{(t^2 + t\sqrt{v_2} + v_2)^{\varepsilon + 1}} \times$$

$$\times \exp\left(\zeta \operatorname{arc\,tg} \frac{2t + \sqrt{v_2}}{\sqrt{3 v_2}} - 2\frac{\psi v_2^{1.5}}{g U_{1\infty}} t\right); \quad (2.6)$$

$R_2 < 1$

$$f_2 = C u_2 \left(\frac{w - u_2}{v_2} - 1\right)^{3\varepsilon - 1} \exp\left(\frac{\psi v_2}{g U_{1\infty}} u_2\right),$$

$$t^2 = w - u_2, \quad \varepsilon = \frac{\psi v_2}{3 g U_{1\infty}} (w - U_{1\infty} - v_2),$$

$$\zeta = \frac{2\psi v_2}{\sqrt{3}\, g U_{1\infty}} (w - U_{1\infty} + v_2). \quad (2.7)$$

The solution of (2.5) exists in the range $(-\infty, u_{2\max}^{(\infty)})$, in which $u_{2\max}^{(\infty)} = w - v_2$; but arguments analogous to those of §1 allow us to show that the velocity of the small particles in the stabilized section cannot be less than $U_{1\infty}$, and so f_2 differs from zero in the range $(U_{1\infty}, u_{2\max}^{(\infty)})$ and is identically zero for other values of u_2.

It follows from (2.6) and (2.7) that the form of f_2 is dependent on ε. For ε > 1/3 ($R_2 < 1$) and for ε > 1/2 (R_2 in range 13 to 800), $f_2(u_{2\max}^{(\infty)}) = 0$; but $f_2(u_{2\max}^{(\infty)}) = \infty$ (Fig. 2) if ε is less than these critical values.

After transformation, the expression for ε for $R_2 < 1$ can be put as

$$\varepsilon = \frac{1}{3}\left[\frac{3}{2}\frac{(\delta_1 + \delta_2)^2}{\delta_1^3}\frac{w\beta_1'}{U_{1\infty}}(u_{2\max}^{(\infty)} - U_{1\infty})^2\right]\frac{1}{(a_2)_{u_2 = U_{1\infty}}}. \quad (2.8)$$

Here the expression in brackets is the mean value of the acceleration equivalent to the action of the large particles on the small ones moving at speed $u_{2\max}^{(\infty)}$ [1], so ε for $R_2 < 1$ has a definite physical significance: it is proportional to the ratio of the total accelerations for small particles with the maximum and minimum velocities, respectively. Also, ε is uniquely determined by that ratio for other ranges in R_2. It follows from (2.3) that ε increases with the concentration of large particles for given δ_1, δ_2, and β_2'.

Equation (2.4) for the stabilized part of the flow is a differential equation with advanced argument. It is readily seen that for

$$u_2 > u_2^* = {}^1/_2\,[(1-k)\,u_{2\max}^{(\infty)} + (1+k)\,U_{1\infty}] \tag{2.9}$$

the argument exceeds $u_{2\max}^{(\infty)}$; so (2.4) becomes (2.5) in the range $(u_2^*, u_{2\max}^{(\infty)})$ which is the initial manifold for (2.4), while the values of f_2 in that range (i.e., the initial function) may be found as indicated above.

The values of f_2 in the range $(U_{1\infty}, u_2^*)$ may be found by successive integration. Substitution for the initial function in (2.4) gives a differential equation in the range (u_2^{**}, u_2^*), in which

$$u_2^{**} = {}^1/_2\,[(1-k)\,u_2^* + (1+k)\,U_{1\infty}].$$

Substitution of this solution into (2.4) gives us the equation for the next range, and so on.

If $k \neq 1$, f_2 also differs from zero in the range $(U_{1\infty}, u_{2\max}^{(\infty)})$.

We can determine $U_{1\infty}$ as follows. The following is the equation of motion for the large particles in the stabilized part, subject to the above assumptions [1, 3]:

$$g\left[\left(\frac{w-U_{1\infty}}{v_1}\right)^{n_1} - 1\right] + a_{1c} = 0\,, \tag{2.10}$$

in which n_1 is dependent on the range in R_1 [3], while a_{1c} is a continuously acting force (per unit particle mass) equivalent to the action of the small particles on large ones during collisions, which can [1] be put as

$$a_{1c} = \frac{3}{4}\,(1+k)\,w\,\frac{(\delta_1+\delta_2)^2}{\delta_1^3}\int\limits_{U_{1\infty}}^{u_{2\max}^{(\infty)}}\frac{(u_2-U_{1\infty})^2}{u_2}\,f_2\,du_2\,. \tag{2.11}$$

We determine f_2 for several values of $U_{1\infty}$, and then graphs of $a_{1c} = a_{1c}(U_{1\infty})$ are drawn up in accordance with (2.10) and (2.11); the abscissa of the point of intersection is the desired value of $U_{1\infty}$.

Equations (2.4) and (2.5) have also been solved by the grid method for the acceleration section, with the variation in U_1 along the flow given [1, 3] by

$$U_1\frac{dU_1}{dL} = g\left[\left(\frac{w-U_1}{v_1}\right)^{n_1} - 1\right] + a_{1c}\,. \tag{2.12}$$

The following example was computed for the stabilized part of the flow for a material consisting of two fractions:

$$\delta_1 = 5\ \mathrm{mm}, \quad \delta_2 = 0.5\ \mathrm{mm}, \quad w = 20\ \mathrm{m/sec}\,,$$
$$\beta_1' = (0.738\text{–}31.2)\cdot 10^{-3}\ \mathrm{m^3/m^3}, \quad v_1 = 12.48\ \mathrm{m/sec}\,,$$
$$v_2 = 1.86\ \mathrm{m/sec}, \quad \beta_2' = 0.738\ \mathrm{m^3/m^3}\,,$$

and the resulting distributions are shown in Fig. 2, in which

curve	1	2	3	4	5	6
k	1	1	1	1	1	0
$\varepsilon =$	0.141	0.397	0.513	1.284	13.45	0.968

The mean speeds of the small particles are determined from the conditions for averaging with respect to the true bulk concentration:

$$\langle u_2\rangle = \beta_2'\left(\int\limits_{u_{1\infty}}^{u_{2\max}^{(\infty)}}\frac{f_2}{u_2}\,du_2\right)^{-1}. \tag{2.13}$$

The results showed that the approximate method of [1] for this problem gives underestimates of the velocities of both fractions, but the error of the method is only 8–10% in these examples.

§3. Consider a dispersed material with a continuous particle-size distribution characterized by a function x:

$$d\beta' = x(\delta)\,d\delta\,. \tag{3.1}$$

The following function F is the distribution by velocity and size:

$$d^2\beta' = F(\delta, u, L)\,d\delta\,du\,. \tag{3.2}$$

The general form of the equation for F does not differ from (1.2). To find $\partial_e F/\partial\tau$ we consider groups A and B of particles having size ranges δ_1, $d\delta_1$ and δ_2, $d\delta_2$ and velocity ranges u_1, du_1 and u_2, du_2, respectively, in section L. The concentration* of group-A particles is as follows:

$$F(\delta_1, u_1)\;d\delta_1 du_1 d\tau/\Delta\tau_{12}\,, \tag{3.3}$$

and these particles collide with group-B particles in time $d\tau$; here $\Delta\tau_{12}$ is defined as in §1. Transformation and integration of (3.3) gives the following expression for the total loss of group-A particles as a result of collisions with all particles in the flow:

$$\frac{3}{2}\,wF(\delta_1, u_1)\,d\delta_1\,du_1\,d\tau \times$$
$$\times\int\limits_{\delta_{\min}}^{\delta_{\max}}\int\limits_{u_{\min}}^{u_{\max}}\frac{(\delta_1+\delta_2)^2}{\delta_2^3}\frac{|u_2-u_1|}{u_2}\,F(\delta_2, u_2)\,du_2\,d\delta_2\,. \tag{3.4}$$

Similarly, the gain in the concentration of group-A particles by collision is

$$\frac{3}{2}\,wd\delta_1\,du_1\,d\tau\int\limits_{\delta_{\min}}^{\delta_{\max}}\int\limits_{u_{\min}}^{u_{\max}}\frac{(\delta_1+\delta_2)^2}{\delta_2^3}\,\sigma_3^2\,\frac{|u_2-u_1|}{u_2}\times$$
$$\times\,F(\delta_1, \sigma_3 u_1 - \sigma_4 u_2)\,F(\delta_2, u_2)\,du_2\,d\delta_2 \tag{3.5}$$

Then (1.2), (3.4), and (3.5) give

$$u_1\frac{\partial F(\delta_1, u_1)}{\partial L} + a_1\frac{\partial F(\delta_1, u_1)}{\partial u_1} + u_1F(\delta_1, u_1)\,\frac{d}{du_1}\left(\frac{a_1}{u_1}\right) =$$
$$= \frac{3}{2}\,w\int\limits_{\delta_{\min}}^{\delta_{\max}}\int\limits_{u_{\min}}^{u_{\max}}\frac{(\delta_1+\delta_2)^2}{\delta_2^3}\frac{|u_2-u_1|}{u_2}\,[\sigma_3^2F(\delta_1, \sigma_3 u_1 -$$
$$-\,\sigma_4 u_2) - F(\delta_1, u_1)]\,F(\delta_2, u_2)\,du_2\,d\delta_2\,. \tag{3.6}$$

It follows from (3.1) and (3.2) that the solution to (3.6) must obey the normalization condition

$$\int\limits_{u_{\min}}^{u_{\max}} F(\delta, u)\,du = x(\delta) \tag{3.7}$$

In the stabilized part, $u_{\max}^{(\infty)} = w - v_{\min}$ is the maximum velocity of the particles of minimum size, while $u_{\min}^{(\infty)} = w - v_{\max}$ is the minimum velocity of the largest particles. As in §1, it can be shown that:

1) F in the stabilized part differs from zero in a rectangle whose vertices are

$$(\delta_{\min}, u_{\min}^{(\infty)}),\ (\delta_{\min}, u_{\max}^{(\infty)}),\ (\delta_{\max}, u_{\min}^{(\infty)}),\ (\delta_{\max}, u_{\max}^{(\infty)})\,.$$

*For brevity, $F(\delta, u)$ is written in place of $F(\delta, u, L)$.

2) If the velocities in the inlet section take values within a rectangle whose sides are parallel to the axes of δ and u, the same form for the region $F \neq 0$ persists in all sections of the acceleration part, with $u_{max} = u_{max}(L)$ and $u_{min} = u_{min}(L)$ defined by the equations of motion for a monodisperse material [3], respectively, for the particles of minimum and maximum size.

Equation (3.6) may also be solved by the grid method.

REFERENCES

1. G. L. Babukha and A. A. Shraiber, "Effects of collisions between particles in a polydisperse material on motion and interphase heat transfer in a vertical two-phase flow," PMTF [Journal of Applied Mechanics and Technical Physics], no. 4, 1966.

2. S. Chapman and T. Cowling, The Mathematical Theory of Inhomogeneous Gases [Russian translation], Izd. inost. lit., 1960.

3. G. L. Babukha and A. A. Shraiber, "Concentration change in a disperse material along a two-phase flow," collection: Flows of Liquids and Gases [in Russian], Izd. Naukova dumka, 1965.

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